Decision Diagram-Based Exact Optimization for Assembly Line Balancing with Human-Robot Collaboration

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Abstract

We study the Type-I assembly line balancing problem with human-robot collaboration (Type-I ALBP-HRC), which jointly requires task-to-station assignment, robot allocation, execution-mode selection (human, robot, or collaborative), and intra-station scheduling under precedence constraints. We develop and compare two independent exact approaches for this problem: a constraint programming (CP) model exploiting interval-based scheduling constraints and a dynamic programming (DP) approach built on a three-level nested dynamic program. Both approaches are benchmarked against a MIP reference model on standard instances. Results show that DP-based engines are strongly competitive for proof-oriented exact search, while CP methods provide tight optimality gaps on hard, large instances within time limits.

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1 Introduction

The assembly line balancing problem with human-robot collaboration (ALBP-HRC) extends classical line balancing by coupling assignment decisions with mode-dependent station scheduling. Recent surveys show strong practical interest in this problem class and highlight the lack of scalable exact approaches for Type-I settings [4, 8]. Given a precedence directed acyclic graph (DAG), cycle time $\mathcal{C}$, and robot budget Q, one must minimize the number of stations while deciding, for each task, its station assignment and execution mode (human, robot, or collaborative).

The challenge is structural: assignment, robot placement, and sequencing are tightly interdependent. A task-to-station assignment that appears feasible can become infeasible once resource disjunctions and mode-dependent durations are enforced within each station. This paper proposes two independent exact approaches to this problem, evaluated separately against common baselines.

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2 Modeling and Exact Approach

Let $T = \{1, \ldots, n\}$ be the task set and $G = (T, E)$ the precedence graph. For each task i, durations t_{iH}, t_{iR}, and t_{iC} are mode-dependent, with robot and collaborative modes available only at robot-equipped stations. A solution is feasible if all precedence constraints hold, every station schedule satisfies a makespan $\leq \mathcal{C}$, and at most Q stations use robots.

2.1 CP model.

The Type-I ALBP-HRC is encoded as a constraint satisfaction problem on a global timeline, where station k occupies the time window $[(k-1) \cdot \mathcal{C},\ k \cdot \mathcal{C}]$. It keeps the high-level timeline structure, but expresses robot feasibility primarily through a `alwaysIn` cumulative step-function profile [6]. This yields a compact containment model in which robot/collaboration operations are forced to occur within active robot windows while preserving disjunctive filtering with the `NoOverlap` constraint. The optional interval variables, the `alternative` constraint used to enforce one selected mode per task, and the `NoOverlap` constraints give strong propagation on cycle-time feasibility and resource non-overlap. A failure-directed search strategy is used to explore the search space [7].

2.2 DP model.

We formulate the problem as a three-level nested dynamic program (DP). The outer problem, denoted HRCPS, assigns tasks to stations and allocates robots for human-robot collaboration under precedence and station-control constraints; the intermediate problem, denoted HRCP, checks single-station feasibility under precedence constraints; the inner problem, denoted HRC, provides a fast checker for single-station feasibility with relaxed precedence. For each level, we specify a state, transitions, and a transition function. From the DP model, a decision diagram (DD) is compiled, and the resulting search space is explored with three search engines: Branch and Bound (B&B), Anytime Column Search (ACS), and A*. For each level, lower bounds and dominance relations are defined to reduce the search space, following the DDOLib framework [5]. Bounded-width multivalued decision diagrams (MDDs) with specific relaxation operators are compiled for the outer DP problem to drive branch-and-bound search [1, 2, 3]. An infeasibility cache and a symmetry-breaking rule are incorporated to focus the search without sacrificing exactness.

3 Computational Summary

We evaluate seven exact configurations on standard Type-I ALBP-HRC benchmarks: one MIP model (Gurobi), three CP solvers (OR-Tools CP-SAT, OptalCP, MaxiCP), and three DDOLib search engines (DDO, ACS, A*). Each configuration is run independently; no information is shared across paradigms during search. Metrics include the number of instances solved to optimality, proof times, and optimality gaps within a fixed time limit.

Results reveal complementary strengths. CP solvers tend to find strong feasible solutions quickly and deliver tight gaps on large instances within the time limit. DDO-based search is particularly effective at certifying optimality on medium instances where the DP structure is well exploited. Neither paradigm dominates uniformly, which motivates reporting both as distinct contributions.

4 Conclusions

This work presents two independent exact approaches for the Type-I ALBP-HRC: a CP model exploiting scheduling constraints and a DP model built on a nested DP. Both are evaluated against a MIP baseline on benchmark instances. The results confirm that CP and DP address the problem from complementary angles, and that both are viable exact methods for collaborative assembly line balancing.

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